Multifractal analysis of Hg pore size distributions of tectonically deformed coals

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A B S T R A C T
To get deep insight into the variability and heterogeneity of pore size distributions (PSDs) in different tectonically deformed coals (TDCs) (mean maximum vitrinite reflectance \( R_{\text{max}} \) ranging from 1.720% to 1.857%) collected from Hancheng Mine, Weibei coalfield, the multifractal analysis using generalized dimensions \( D_q \) was employed to study PSDs of TDCs based on mercury intrusion porosimetry. The mercury intrusion data under pressure over 10 MPa was corrected in combination with gas adsorption data. The results show that, for the test coals, tectonic deformation (TD) mainly increases seepage-porosity while maceral composition (MC) is the main control on adsorption-porosity. The variations of \( D_q \) versus \( q \) show that the PSDs of TDCs exhibit multifractal behavior, however, differ in their multifractality suggested by the extracted parameters from PSDs including information dimension \( D_1 \), the Hurst exponent \( H \), the width of the right side \( D_{10} \) and the left side \( D_{-10} \) of \( D_q \) spectra. TD and MC have great influence on the variability and heterogeneity in the inner distribution of seepage-porosity and adsorption-porosity respectively, as demonstrated by the change of \( D_1 \), \( H \) and \( D_{10} \) for moment \( q > 0 \) and \( D_{-10} \) for moment \( q < 0 \). Both \( D_1 \) and \( H \) increase with TD but \( D_{10} \) shows a reverse trend. Consequently, TD leads to narrower distribution with higher fluctuation, lower pore connectivity and greater complexity in the distribution of seepage-pores, which may be due to the presence and uneven distribution of microfracture and inter-granular pores formed through deformation process. Increasing vitrinite and decreasing inertinite contribute to a heterogeneous distribution of adsorption-porosity, as observed by the high correlation between \( D_{10} \) and MC. The aforementioned results confirm that the multifractal approach is useful to characterize the internal heterogeneity of PSDs and to detailly distinguish between PSDs of TDCs.

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1. Introduction

The pore size distributions (PSDs) of coals have gained wide attention since they have a significant influence on the success of exploiting coalbed methane (CBM) from wells (Clarkson and Bustin, 1999; Zhang et al., 2013a, 2013b). The pore properties are related to the complex physical and chemical structures of coals (Bustin et al., 1995; Firouzi et al., 2014; Xu et al., 2014). The flow property and storage capacity of gas in coals vary due to the heterogeneous textural structure of coals.

Tectonically deformed coals (TDCs) were formed under tectonic evolutionary histories (Liu et al., 2000), and thus it results in strong transformation of coal structure and various TDCs, which might induce changes in physical properties of CBM reservoirs (Beamish and Crosdale, 1998; Cao et al., 2003). Under tectonic deformation, the PSDs have changed, resulting in different gas transportation and adsorption behavior and providing new insight into CBM recovery in deformed coalbeds (Hou et al., 2012; Li et al., 2003). Therefore, a full elaboration of the PSDs of TDCs will be favorable for CBM recovery and coal and gas outbursts prediction.

Various methods were used to study PSDs of coals. Statistical descriptions of the PSDs can be inferred indirectly from mercury intrusion porosimetry (MIP) (Yao and Liu, 2012; Zou et al., 2013), small-angle scattering (Radlińska et al., 2004) and gas adsorption analyses (Bae et al., 2009). Direct observational methods such as scanning electron microscopy (SEM) (Giffin et al., 2013), microfocus X-ray computed tomography (μCT) (Golab et al., 2013; Kumar et al., 2011) and atomic force microscope (AFM) (Lawrie et al., 1997; Wu et al., 2014) permit quantification of pore properties including porosity, pore size and connectivity on two- and three-dimensional views. Among these, MIP is frequently used to estimate PSDs and has been used to characterize pore size of TDCs (Qu et al., 2010). The MIP method is useful for characterizing porosity distribution with pore size above 3 nm (Gan et al., 1972; Ritter and Drake, 1945), but the high pressure (\( P > 10 \) MPa) may damage narrow pores or compress the coal matrix, and the
reduction in bulk volume of the sample could occur (Friesen and Mikula, 1988). With the applied pressure increasing, the simultaneous compression and pore-filling occur during mercury intrusion (Li et al., 1999). Thus, prior to characterization of the PSDs by using MIP data, the effects of coal compressibility at pressure over 10 MPa should be carefully examined and corrected, and then the corrected Hg pore volume can be used to characterize PSDs for coals (Cai et al., 2013; Gökhan Şenel et al., 2001). However, the mercury intrusion data correction has not been carried out in the evaluation of PSDs related to TDCs, which would lead to an erroneous result for pore volume evaluation without compressibility correction. Furthermore, the compressibility of coal used to adjust the Hg pore volume is difficult to determine using the MIP method alone (Friesen and Mikula, 1988). N₂ adsorption is useful for characterizing porosity in the pore size range of 2–300 nm (Gan et al., 1972; Unsworth et al., 1989), and CO₂ adsorption works well for characterizing micro–porosity (<2 nm) (Liu and Wilcox, 2013; Mastalerz et al., 2008). In this work, N₂ adsorption data were used to characterize the pore-filling volume measured by MIP in the high pressure interval. In combination with MIP data and N₂ and CO₂ adsorption data, the compressibility of seven typical TDCs was determined and the Hg pore volume was adjusted based on the work of Li et al. (1999). The PSDs related to coal deformation structures after Hg data correction were further discussed.

The pore sizes of coals are not uniformly distributed in space, which makes it difficult to characterize the complexity of PSDs by traditional Euclidean geometry. In that case, fractals, initially proposed by Mandelbrot (1967), have been widely used to quantitatively characterize physical properties of spatially non-uniform systems and one parameter, the fractal dimension, D, has been introduced to describe the irregular distribution of pore sizes of coals (Gauden et al., 2001; Yao et al., 2009a). However, the profiles of the distribution of pore sizes of coals often show “fluctuations” and “jumps” at different pore size intervals and, in general, types of erratic variation or local variation occur in the inner distribution of pore sizes which cannot be explained by a single-scale (monofractal) analysis or a single fractal dimension. In fact, for irregular distribution of pore sizes of coals, a single fractal dimension would describe the irregularity within limited size intervals, that is, different pore size intervals would show different types of self-similarity (Friesen and Mikula, 1987; Li et al., 1999). These characteristics can also be found in many non-uniform PSDs in nature which can be described as multifractal structure (Muller and McCauley, 1992). These complex distributions can be attributed to the underlying nonlinear dynamics, as superposition of different processes acting simultaneously along a wide range of length scale (Peitgen et al., 1992). Studies have shown that the variation and distribution of pore sizes for porous media can be assigned to an underlying nonlinear dynamics system, and multifractal analysis seems to be an appropriate tool to analyze the inner non-linear variation or heterogeneous distribution of pore sizes for porous media (Caniego et al., 2003; Muller, 1996; San José Martínez et al., 2010).

Multifractals can be treated as an extension of fractals. Indeed, a multifractal structure is considered as a superposition of monofractal structure (Posadas et al., 2003). In comparison of monofractal structure, the multifractal structure can be decomposed into a set of intertwined fractal subsets demonstrated by a hierarchy of scaling exponents that characterize the local variability and heterogeneity of studied variables (Krvavchenko et al., 1999). The multifractal analysis captures the inner variations in a system by resolving local densities (probability) and expresses them by a continuous of fractal dimensions spectrum referred to as singularity spectrum and generalized dimension spectrum (Chhabra et al., 1989). In view of this, the multifractal approach has been successfully applied to characterize the inner variation of PSDs in soil (Montero and Martin, 2003) and sedimentary rocks (Anowitz et al., 2013; Cheng, 1999). Several authors carried out multifractal studies of PSDs of porous material from image analysis. San José Martínez et al. (2010) performed the multifractal analysis of soil macropore structure at horizon scale using X-ray computed tomography (CT) data. It was found that the multifractal approach is an effective tool for parameterizing the spatial heterogeneity of soil macropore structure. Muller and McCauley (1992) first used the idea of multifractal scaling to characterize the PSDs in sedimentary rocks through an optical microscope. They demonstrated how the multifractal scaling of pore space can be used as a tool for rock characterization. Xie et al. (2010) investigated the multifractal analysis of porosity of sedimentary carbonate from two-dimensional environmental scanning electron microscope (ESEM) image. They observed that the reservoir capacity assessment result derived from multifractal analysis is consistent with the field research. Recently, multifractal analysis of PSDs determined by MIP method has been performed (Paz Ferreiro and Vidal Vázquez, 2010). Sanjurjo-Sánchez and Vidal Vázquez (2013) assessed the surface weathering of a granite rock by combination of elemental chemical analysis and multifractal analysis of Hg injection data sets. They found that multifractal parameters of PSDs are suitable indicator for evaluating weathering. From the above literature review, it is found that multifractal analysis of PSDs measured by Hg injection in TDCs has not been performed.

In this work, we use the multifractal approach to investigate the variability and heterogeneity of PSDs in different TDCs with Hg pore volume–size data after compressibility correction and to determine whether multifractal parameters can be used to compare the variability of PSDs in different TDCs with similar coal rank. The quantitative characterization of PSDs based on multifractal analysis may provide relevant information which can be used to improve our understanding how tectonic deformation affects PSDs in coal.

According to the work of Unsworth et al. (1989) and Cai et al. (2013), a combined pore classification from Hodot (1961) and IUPAC (Sing et al., 1985) is employed in this study: microfracture (d > 10,000 nm), macropore (1000 nm < d < 10,000 nm), mesopore (100 nm < d < 1000 nm), transition pore (10 nm < d < 100 nm), micropore (2 nm < d < 10 nm), and super-micropore (d < 2 nm), where d is the pore diameter. Based on the interaction of gas molecular and pore size, macropore and mesopore are called seepage–pore, where gas laminar flow occur during production, while transition pore, micropore and super-micropore are assigned to adsorption–pore in which gas diffusion and physical adsorption occur (Shi and Durucan, 2005). Among these pores, the super-micropore plays a dominant role in gas adsorption (Mastalerz et al., 2008).

2. Geological setting

The Hancheng mining district (HMD) is located in the southeastern margin of Ordos Basin. Structurally, it is a monoclinal structure, northwest trend, southeast dip with 5°–10°. The district experienced three big tectonic events including the Indo-Chinese, Yanshanian and Himalayan periods. Gentle fold with the axis trending EW–NWW formed during the Yanshanian stage. The HMD suffered S–N and SE–NW extension under NWW compression during the Indosinian stage. The fold and thrust faults striking NE formed under intensive SE–NW compression during the Yanshanian stage. The HMD suffered S–N and SE–NW extension under NWW compression during the Himalayan stage (Yao et al., 2009b). The regional structures in HMD possess strong structure in east and south and weak structure in north and west due to the effect of multiple tectonic deformation (Fig. 1a). The strong tectonic deformation is concentrated in the southeastern margin of the district. Two sets of structure are well developed in Hancheng mining area: one set strikes NNE–NE, such as F1 normal fault located in the eastern margin, another set strikes NEE, including Longting, Dongzejun and Longguling tectonic belt from south to north (Fig. 1a).

Based on the distribution of structures in HMD, the dominant structure in Shangshuping (SSP) coalmine located in the north of HMD is compressional fold and the collected samples in SSP are located in the axial part of syncline (Fig. 1a). The structure in Xiangshan (XS) coalmine located in the south of HMD shows somewhat complicated. The small-scale bed-parallel compressional faults are developed and the collected
coal samples in XS lie in these shear zone (Fig. 1b). Additionally, in underground mines, a different degree of tectonic deformation occurs in coal seam due to the uneven distribution of the structure. For example, within the XS coalmine, as shown in Fig. 1d, the coal seam called “soft-coal” band exhibits a change in overall structure and is extensively polished under intensive shear deformation. The small-scale structure, such as small fold and kink band, can be observed. In contrast, under weak shear deformation, coal lithotype and cleat can be distinguished as presented in Fig. 1c. Moreover, tectonically striated fracture (described in later Section 4.1) can be found.

The coal-bearing stratum is mainly composed of Pennsylvanian Taiyuan Formation and Permian Shanxi Formation with a total thickness of 85–175 m and eleven coal seams. The main seams in the Taiyuan Formation are No. 5 and No. 11 and No. 3 is the main seam in the Shanxi Formation. The thicknesses of No. 3, No. 5 and No. 11 are 0–3 m, 0–10 m and 2–6 m, respectively. All the seven typical TDC samples were collected from No. 11 coal seam and were originally in the form of hand specimens taken from the active working faces. The deformation features of the experimental samples will be elaborated later.

3. Coal samples and experiments

The maximum vitrinite reflectance ($R_{o,max}$) measurements of the seven samples were performed following the GB/T 6948-2008 standard. These samples are low volatile bituminous coals with $R_{o,max}$ ranging from 1.720% to 1.857% (Table 1). Vitrinite contents of coals are from 19.5% to 83.7% and inertinite contents vary from 15.7% to 79.5%. Mineral matters vary between 0.4% and 2.6% (Table 1). The coal samples can be divided into vitrinite-rich coals and inertinite-rich coals using the arbitrary boundary of <50% vitrinite, respectively (Unsworth et al., 1989). That is, in addition to sample SSP16 defined as inertinite-rich coals, the rest of the samples are vitrinite-rich coals. The ultimate analysis and proximate analysis of the seven samples were carried out following methods GB/T 476-2008 and GB/T 212-2008, respectively. Results including ash and moisture contents, and carbon and hydrogen contents were given in Table 1.

Mercury injection porosimetry (MIP) analysis was performed using an Autopore 9310 Instrument (Micromeritics, America) from China University of Mining and Technology. The samples for MIP experiments were about a 2 g weight with a block size of 1–2 cm. Prior to each experiment, all the coal samples were dried at 60 °C for 12 h. The dry samples were evacuated from the low-pressure port to <50 μm Hg to remove the residual gas and moisture in the sample. To evaluate the pore diameter using the Washburn equation (Washburn, 1921), the contact angle between mercury and the pore surface of 130°, and the surface tension of 485 dyn/cm were used as suggested by Gan et al. (1972) and Gökhan Şenel et al. (2001). The measurements run up to a pressure of 206.75 MPa, indicating that pore diameter as small as 6 nm can be penetrated. In order to diminish the effects of microfracture and interparticle porosity at low mercury pressure, the pore size data of 10 μm...
Table 1
Deformed feature, petrologic, and chemical analysis of selected coal samples.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Deformed feature</th>
<th>Rm,max (%)</th>
<th>Maceral and mineral (%)</th>
<th>Proximate analysis (% dry)</th>
<th>Ultimate analysis (% dry)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vitrinite</td>
<td>Inertinite</td>
<td>Mineral</td>
</tr>
<tr>
<td>X57</td>
<td>Cataclastic</td>
<td>1.857</td>
<td>53.2</td>
<td>44.8</td>
<td>2.0</td>
</tr>
<tr>
<td>X51</td>
<td>Cataclastic</td>
<td>1.720</td>
<td>80.0</td>
<td>18.2</td>
<td>0.9</td>
</tr>
<tr>
<td>X59</td>
<td>Cataclastic</td>
<td>1.777</td>
<td>93.7</td>
<td>15.7</td>
<td>0.6</td>
</tr>
<tr>
<td>X55</td>
<td>Granulated</td>
<td>1.743</td>
<td>73.6</td>
<td>26.0</td>
<td>0.4</td>
</tr>
<tr>
<td>X56</td>
<td>Granulated</td>
<td>1.838</td>
<td>61.1</td>
<td>36.9</td>
<td>2.0</td>
</tr>
<tr>
<td>X58</td>
<td>Mylonitic</td>
<td>1.813</td>
<td>69.8</td>
<td>29.2</td>
<td>1.0</td>
</tr>
<tr>
<td>SSP16</td>
<td>Mylonitic</td>
<td>1.797</td>
<td>19.5</td>
<td>79.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

(0.12 MPa) was used as the upper limit according to the work of Unsworth et al. (1989).

Scanning electron microscope (SEM) analysis on coal samples was carried out using Quanta-200F field emission electron microscope scanning at China University of Petroleum in Beijing. The SEM technique used in this study enables us to observe the coal microscopic deformation structure and to investigate morphology and sizes of pores and microfractures falling into macropore range (mainly >0.1 μm). In order to well analyze the microstructure of different TDCs, for each sample, three patches with a block size of 1 cm³ in different positions were chosen and the relatively flat coal surface of each sample was selected for SEM observation. For the examination, the selected samples were posted on the sample stage, and the attachment on the samples surface was cleaned using ear washing bulb, then the sample surfaces were sputter-coated with gold-palladium for SEM investigation.

True densities of the samples were measured by helium displacement using a Quantachrome UltraPyc 1000 helium pycnometer. N₂ adsorption at 77 K and CO₂ adsorption at 273 K were conducted using an automatic gas adsorption apparatus (NOVA-4200e, Quantachrome) to obtain pore volume in the pore size range of 2 nm to 200 nm and super-micro pore (~2 nm) volume, respectively. Prior to the measurement, the sample was dried at 105 °C overnight in a vacuum oven. For three techniques due to different assumptions of pore models and experimental artifacts specific to each method. In spite of these difficulties, it is an effective method to gain insight into the distribution of pores ranging from <2 nm to >10,000 nm in coals (Clarkson and Bustin, 1999; Gan et al., 1972; Unsworth et al., 1989).

4. Analysis

4.1. Mercury porosimetry analysis

Some previous studies found that coal compressibility has an obvious effect on MIP results when pressure exceeds 10 MPa (Friesen and Mikula, 1987). So, we calculated coal compressibility of the studied samples with pressure over 10 MPa. Coal compressibility can be defined as (Li et al., 1999):

\[ k_c = \frac{dV_c}{dP} \]

where \( dV_c/dP \) represents the coal matrix volume change as a function of pressure, and \( V_c \) is the coal matrix volume. Since coals contain a large amount of mesopores and micropores, some of the pores could not be penetrated by mercury even at the highest pressure applied, and \( V_c \) in Eq. (1) comprises some unfilled pores (Friesen and Mikula, 1988; Li et al., 1999).

For a compressible solid (Li et al., 1999):

\[ \Delta V_{obs} = \Delta V_{trans} + \Delta V_c \]

where \( \Delta V_{obs}, \Delta V_{trans}, \) and \( \Delta V_c \) are the changes of observed mercury volume, pore-filling volume, and coal matrix compression volume, respectively.

A good linear relation can be observed in the plots of the observed mercury intrusion volume versus pressure with \( P > 10 \) MPa for different TDCs (Fig. 2a). A similar phenomenon also can be found in the work of Toda and Toyoda (1972) and Guo et al. (2014). Therefore, \( \Delta V_{obs}/\Delta P \)

Table 2
Parameters obtained from MIP, N₂ and CO₂ adsorption analysis and true density.

| Sample ID | \( \rho_V \) g/cm³ | \( k_c \times 10^{-10} \) m³/N | \( V_{mic} \) cm³/g | \( V_{mic} \) cm³/g | \( V_{ads-mic} \) cm³/g | \( V_{ads-super} \) cm³/g |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| X57       | 1.507           | 0.262           | 0.0236          | 0.0142          | 0.0037          | 0.037           | 0.030           | 0.676           | 0.030           | 0.018       |
| X51       | 1.505           | 0.715           | 0.0281          | 0.0175          | 0.0040          | 0.0037          | 0.037           | 0.737           | 0.050           | 0.028       |
| X59       | 1.496           | 0.747           | 0.0322          | 0.0211          | 0.0069          | 0.0044          | 0.0033          | 0.927           | 0.053           | 0.020       |
| X55       | 1.514           | 0.873           | 0.1053          | 0.0926          | 0.0105          | 0.0272          | 0.0477          | 0.868           | 0.059           | 0.018       |
| X56       | 1.476           | 0.725           | 0.0877          | 0.0768          | 0.0079          | 0.0290          | 0.0330          | 0.727           | 0.050           | 0.027       |
| X58       | 1.489           | 0.914           | 0.0805          | 0.0758          | 0.0095          | 0.0185          | 0.0411          | 0.742           | 0.051           | 0.028       |
| SSP16     | 1.559           | 0.886           | 0.0796          | 0.0672          | 0.0107          | 0.0189          | 0.0311          | 0.964           | 0.025           | 0.014       |

\( V_{mic} \), original maximum mercury pore volume obtained from MIP; \( V_{mic}^{*} \), maximum mercury pore volume with compressibility correction; \( V_{ads-mic} \), super-micro pore volume determined from mercury intrusion (<10,000 nm in diameter); \( V_{ads-super} \), microfracture pore volume determined from mercury intrusion (10,000–10,000 nm in diameter); \( V_{mic} \), corrected macropore volume (100–1000 nm in diameter); \( V_{ads} \), corrected mesopore volume (100–1000 nm in diameter); \( V_{mic}^{*} \), BET specific surface area; \( V_{ads} \), the BJH total pore volume; \( V_{mic} \), transition pore volume (10–100 nm in diameter) of BJH pore volume; \( V_{mic} \), micropore volume (2–10 nm in diameter) of BJH pore volume; \( V_{mic} \), Dubinin–Radushkevich (DR) specific surface area; \( V_{super-mic} \), super-micro pore volume (<2 nm in diameter); \( V_{ads} \), volume of adsorbed amount in super-micro pore.
can be assumed to be constant \( \beta \) under \( P > 10 \text{ MPa} \), that is, \( \Delta V_c / \Delta P \) can be obtained approximately by (Li et al., 1999):

\[
\frac{\Delta V_c}{\Delta P} = \beta \frac{100 \text{nm}}{\sum \Delta V_p}
\]

where the sum of the volume of pores of 6–100 nm (equivalent pressure varies from 206.75 MPa to 10.99 MPa) can be obtained from \( N_2 \) adsorption data (Table 3). Constant \( \Delta V_{\text{obs}} / \Delta P \) or \( \Delta V_c / \Delta P \) is valid only if the pores included in the sample remain unchanged during compression (Li et al., 1999). Assuming \( \Delta V_c / \Delta P \) is independent on pressure, and replacing \( \Delta V_c / \Delta P \) by \( \Delta V_c / \Delta P \), the compressibility of the seven coal samples can be obtained by Eq. (1). Since the minimum pore size probed is limited by the highest pressure available with mercury instrument, it is reasonable that the true solid volume and micropores are compressed simultaneously (Li et al., 1999). Thus, \( V_c \) was determined from the sample true density plus the micropores obtained from \( \text{CO}_2 \) adsorption, and the mesopores below 6 nm from \( N_2 \) adsorption (Li et al., 1999).

In Table 2, compressibility values, \( \Delta c / \Delta p \), are in agreement with the values reported in the literature (Guo et al., 2014; Toda and Toyoda, 1972).

The detailed discussion about Hg pore volume calibration method can be found in Li et al. (1999). The original observed mercury intrusion data and the corrected mercury intrusion data with compressibility correction were presented in Fig. 2b. It can be seen that the differences between original and corrected data indicate that coal compressibility has an obvious effect on Hg pore volume especially for pressure exceeding 20 MPa, which is compatible with the results obtained by Suuberg et al. (1995). Table 3 listed the pore volume in the pore size range of 6–100 nm where mercury intrusion and \( N_2 \) adsorption method overlap. As can be seen in Table 3, the difference in pore volume from the data of original MIP and \( N_2 \) adsorption on average accounts for 14.75% of total pore volume, while they are less than 5% from the data of corrected mercury and \( N_2 \) adsorption. The result coincides with that observed by Li et al. (1999). Taking into account of the effect due to coal compressibility, the corrected MIP data can generate multifractal parameters.

4.2. Multifractal analysis of Hg PSDs of coals

The multifractal analysis of PSDs can be performed through two-dimensional image analysis at a plane (Posadas et al., 2003) or by one-dimensional Hg injection at a size interval (Paz Ferreiro and Vidal Vázquez, 2010). Vidal Vázquez et al. (2008) have applied multifractal approach to assess one-dimensional PSDs estimated by Hg injection in soil. So, in our study, the multifractal analysis of one-dimensional Hg PSDs with pore volume–size data in coals is performed based on the work of Vidal Vázquez et al. (2008). The heterogeneity of PSDs can be characterized by the singularity spectrum or equivalently by the generalized dimensions using multifractal approach (Caniego et al., 2001; Muller and McCauley, 1992). However, the generalized dimensions have an easier handling and interpretation than singularity spectrum as suggested by Muller (1996) and Caniego et al. (2003). Thus, we use the generalized dimensions to assess the variability and heterogeneity of Hg PSDs.

To execute multifractal analysis of Hg PSDs for porous media supported on an interval \( I = [a, b] \), a set of different boxes or subintervals of \( I \) with equal length \( \varepsilon \) is required (Vidal Vázquez et al., 2008). Mostly, the dyadic scaling down is widely used to partition the support \( I \) into a number of boxes \( N(\varepsilon) = 2^k \) of box size, \( \varepsilon = L \times 2^{-k} \) in stages \((k = 0, 1, 2, 3,...)\), where \( L \) is the length of the interval \( I \) of pore size (Caniego et al., 2003). In these boxes or subintervals, the respective measures \( p_k(\varepsilon) \) can be calculated from the available data.

In our case, the interval \( I \) of pore size that varied from 0.006 \( \mu \text{m} \) to 10 \( \mu \text{m} \) including 56 subintervals \( I_i = [a_i, a_i + 1] \) has been considered. The measure \( p_k(\varepsilon) \) in each subinterval is the relative Hg pore volume data, \( V_i \). In other words, the Hg pore volume data, \( V_i \), are normalized, \( V_i = \varepsilon / \sum_i 2^k \). Implementation of multifractal scaling for PSDs in the interval \( I \) of pore sizes requires dyadic partitions of \( I \) into boxes or subintervals of equal length (Monteiro and Martin, 2003). Thus, a rescaling of interval \( I \) of pore sizes is necessary. The most common way is logarithmic transformation. Under the transformation, the normalized pore size changes to \( A_i = \log(a_i/a_0) \) (\( i = 1, 2, 3,..., 56 \)), and a new dimensionless interval \( J = [0, 3.18] \) with subinterval of equal length, \( J_i = [A_i, A_i + 1] \) (\( i = 1, 2, 3,..., 56 \)) is obtained. Then a number \( N(\varepsilon) = 2^k \) of boxes or subintervals of equal size \( \varepsilon = L \times 2^{-k} \) for \( k = 0 \) to 5 will be then consecutively

### Table 3

Comparison of pore volume in the pore size range of 6–100 nm estimated by Hg injection and \( N_2 \) adsorption.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>V (pore volume in the pore size range of 6–100 nm)</th>
<th>( a )</th>
<th>( b )</th>
<th>( N_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>XS7</td>
<td>0.0116</td>
<td>0.0033</td>
<td>0.0020</td>
<td></td>
</tr>
<tr>
<td>XS1</td>
<td>0.0159</td>
<td>0.0054</td>
<td>0.0031</td>
<td></td>
</tr>
<tr>
<td>XS9</td>
<td>0.0169</td>
<td>0.0056</td>
<td>0.0034</td>
<td></td>
</tr>
<tr>
<td>XS5</td>
<td>0.0191</td>
<td>0.0084</td>
<td>0.0021</td>
<td></td>
</tr>
<tr>
<td>XS6</td>
<td>0.0170</td>
<td>0.0072</td>
<td>0.0031</td>
<td></td>
</tr>
<tr>
<td>XS8</td>
<td>0.0195</td>
<td>0.0068</td>
<td>0.0031</td>
<td></td>
</tr>
<tr>
<td>SSPI6</td>
<td>0.0181</td>
<td>0.0066</td>
<td>0.0016</td>
<td></td>
</tr>
</tbody>
</table>

\( a \), original mercury pore volume with pore size ranging from 6 nm to 100 nm; \( b \), corrected mercury pore volume with pore size ranging from 6 nm to 100 nm.
generated in the normalized interval $I$ of pore sizes. In order to ensure that each box has some concentration of pore volume, the maximum value for $k$ is designated as 5 in this study. For any box or subinterval $J_i = [A_i, B_i]$, the measure $p_i(\varepsilon)$ can be obtained by adding all concentrations $V_i$ with normalized pore sizes less than $B_i$ and greater than $A_i$.

The probability density distribution of $p_i$ related to Hg porosity is then analyzed by the partition function, $\chi(q, \varepsilon)$ (Chhabra et al., 1989), which can be calculated from pore volume–size statistics by using

$$
\chi(q, \varepsilon) = \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon)
$$

(4)

where the moment order $q$ is a real number, varying from $-\infty$ to $+\infty$. It serves as a “microscope” for exploring different regions of the pore volume–size distributions. For $q \ll 1$, the value of $\chi(q, \varepsilon)$ is largely determined by small $p_i(\varepsilon)$ data. For $q \gg 1$, the large $p_i(\varepsilon)$ data contribute most to $\chi(q, \varepsilon)$. Thus the variation of $\chi(q, \varepsilon)$ with different $q$ splits the measure into subsets dominated by locally high or small concentration of porosity. Then, the distribution of local porosity can be represented by a series of generalized dimensions, $D_q$ (Muller, 1996), defined by Grassberger and Procaccia (1983), based on the work of Rényi (1955) for all $D_q \neq 1$, as follows:

$$
D_q = \lim_{\varepsilon \to 0} \frac{\log \sum_{i=1}^{N(\varepsilon)} p_i^q(\varepsilon)}{\log \varepsilon}
$$

(5)

For $q > 0$, $D_q$ emphasizes areas with a high concentration of porosity. For $q < 0$, $D_q$ amplifies areas of low concentration (Caniego et al., 2003; Muller, 1996). For $q = 1$, Eq. (5) is uncertain. In this case, $D_1$ is evaluated by L'Hôpital rule:

$$
D_1 = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{N(\varepsilon)} p_i(1, \varepsilon) \log p_i(1, \varepsilon)}{\log \varepsilon}
$$

(6)

Then, the set of points $(q, D_q)$ generates a curve that describes the Rényi or $D_q$ spectrum of the measure $p$. The $D_q$ value at $q = 0, q = 1$ and $q = 2$ corresponds to the capacity dimension $D_0$, information dimension $D_1$, correlation dimension $D_2$, respectively. It is noted that for one-dimensional distribution, the value of $D_0 = 1$ when all the subintervals contain some pore volume data (San José Martínez et al., 2010). As proposed by Riedi et al. (1999), the correlation dimension $D_2$ also can be written as

$$
D_2 = 2H - 1
$$

(7)

where $H$ is the Hurst exponent, varying from 0.5 to 1, usually associated with positive autocorrelation or long-range spatial variation (Feder, 1988). Then the autocorrelation in long-range spatial variations of porosity along the pore size intervals can be parameterized by the Hurst exponent (San José Martínez et al., 2010). In the case of monofractal distribution, the $D_0$ spectrum is a horizontal line, i.e., $D_0 = D_1 = D_2$. For a multifractal structure, this spectrum is a monotone decreasing function of $q$ with a sigmoidal shape and the values of $D_0$, $D_1$, and $D_2$ ranks as $D_0 > D_1 > D_2$ (Caniego et al., 2003).

5. Results and discussions

5.1. Structural characterization

Based on hand specimen examination and SEM observation, the degree of tectonic deformation is evaluated by the destruction of coal primary structure (e.g. banded structure of maceral composition), particle size and hardgrove grindability index (HGI) of coal, fracture properties and strain markers of deformation structure.

The sample XS7 is dominated by bright lithotype with developed linearization structure of clarain and conchohal fracture. Irregular reticular fractures occur (Fig. 3a), indicating weak brittle failure (Su et al., 2001). “Feather-like” or radial fractures with dull lustre occur on coal surface (Fig. 3b). A similar phenomenon also can be found in the work of Bustin (1982a) and Frodsham and Gayer (1999). They suggested that these “feather-like” fractures may be the result of static and brittle shear failure and they defined it as striated fracture. The coal can be easily split into 2–5 mm-scale particles along the reticular fracture and the HGI is 61. SEM observation shows that the coal displays plate-like or wedge-shaped angular structure due to shearing fracture (Fig. 4a) or tensional fracture (aperture varies from 1–6 μm) (Fig. 4b) and micro-scale interlayer-sliding by fracture with aperture of 3.39 μm (Fig. 4b). The interconnected layer-fracture and cleat (non-tectonic fracture) are favorable for gas flow (Fig. 4c). Micro-scale thrust occurs along a bedding shear zone (Fig. 4c). Angular fragments of coal define the sense of shear as sinistral (Fig. 4d). The bright conchohal fracture and dull tectonically striated fracture are observed by means of SEM observation (Fig. 4e).

The macrolithotype of sample XS1 is mainly semibright with little banded dull. The bedding structure is obvious. The coal is cut by oblique tectonic-fracture and is easily split along these fractures (Fig. 3c). The coal is relatively hard and the HGI is 47. Under SEM, block structure with displacement between blocks occurs along the tectonic fracture (Fig. 4f). The mineral band structure is well preserved (Fig. 4f). The coal is composed of different sized brecceas by shearing fracture and tensional fracture which can be filled by pyrite (Fig. 4g). The conchohal fracture is well developed on coal surface (Fig. 4g). The elongate pores in inerminite-rich layer between vitrinite-rich layer are formed under brittle compression (Fig. 4h).

The macrolithotype of sample XS9 is dominated by semibright and the fracture is mainly tectonic-fracture. Massive small fractures in different directions divide the coal into subangular or lumpy particles with size of approximately 1 cm. Large-scale irregular friction mirror surfaces can be observed (Fig. 3d). The HGI of this coal is 52. Under SEM, the fracture surface is occupied by parallel ridges and grooves (Fig. 4i). The coal shows parallel angular structure by shearing fracture or cleavage (Fig. 4j).

Sample X55 has porphyritic texture with dull lustre. The macrolithotype of porphyritic angular blocks with multidirectional small-scale tectonic fracture is dominated by semibright (Fig. 3e). Irregular friction mirror surfaces with reticular fracture occur along the slip plane (Fig. 3f). The coal is easy to crumble into mm-scale grains and the HGI is 85. Microscopically, the coal is composed of fragments of coal clasts without internal deformation and the size of clasts is mainly less than 20 μm (Fig. 4k). Microfracture and inter-granular pores with aperture mainly less than 5 μm are formed between these coal clasts (Fig. 4l). Polished fractures associated with a thin layer of granular coal beneath the fracture plane are pervasive (Fig. 4l, m), which lies in the observation of Frodsham and Gayer (1999), who suggested that these fractures are the product of dynamic and brittle shear failure. The open and zigzagged fracture with aperture of 5.12 μm (Fig. 4n) and striated fracture (Fig. 4o) also can be found in this coal.

Sample XS6 has similar deformation feature with sample XS5. The bedding structure and non-tectonic fracture are strongly damaged. The coal has lenticular texture with dull lustre (Fig. 3g). The coal is easy to crumble into mm-scale grains and the HGI is 93. SEM observation shows that the coal clasts (size can be less than 2 μm) are well developed (Fig. 4p). Fig. 4q shows that the inerminite is subjected to heterogeneous deformation. To the left is cracked inerminite, while slip plane and thin layers of coal clasts occur under shear stress on the right, showing a mortar texture.

Sample XS8 and Sample SSP16 have similar deformation structure. Both are characterized by mixed petrographic components with dull lustre and strongly wrinkled structure, indicating obvious ductile deformation (Fig. 3h, i) as suggested by Bustin (1982b) and Li (2001). The
5.2. Pore size distributions of different TDCs

Inertinite-rich coals show multi-model distribution of pore sizes and the pore volume in Fig. 5. As can be seen in Fig. 5 and Table 2, all the cataclastic coals show a high frequency of pores at diameter about 740 nm, which falls into the range of super-micropore sorption capacity of granulated and mylonitic coals. The PSDs of cataclastic coals are characterized by multi-modal distribution with a peak at around 740 nm. In contrast, granulated and mylonitic coals exhibit a broader range of pore sizes with a peak at around 30 nm. The super-micropore volume, DR surface area and the greatest amount of pore volume in the pore size range of 30–1000 nm (corresponding pressure ranges from 1.2–0.6 MPa) with the highest frequency of pores at diameter about 740 nm is noticed in granulated and mylonitic coals. The macro- and mesoporosity in cataclastic coals is low (8.14% to 10.56%), but super-micropore approximately accounts for 85.55% of the total porosity (seen in Table 2), which may be caused by maceral composition.

In comparison to cataclastic coals, the macro- and mesopore volumes of granulated and mylonitic coals sharply increase, and the greatest amount of pore volume in the pore size range of 300–1000 nm (corresponding pressure ranges from 1.2–0.6 MPa) with the highest frequency of pores at diameter about 740 nm is noticed in granulated and mylonitic coals. The results indicate that granulated and mylonitic coals possess six to nine times seepage–pore volume (macro- and mesopore volume) than cataclastic coals, which is consistent with the findings of Li et al. (2003), who applied mercury injection to characterize pore structure of TDCs. However, the super-micropore volume, DR surface area and super-micropore sorption capacity of granulated and mylonitic coals are smaller than those of cataclastic coals, and the adsorption pore volume and surface area of vitrinite-rich coals are higher than those of inertinite-rich coals (Table 2). Moreover, it is found from Fig. 2b that the slopes of volume-pressure curves for granulated and mylonitic coals are highly friable and the HGI is 138 for XS8 and 118 for SSP16, respectively. The SEM observations of these two coals are summarized as follows. Kink band with micro-scale fold occurs in inertinite-rich layers (Fig. 4r) and vitrinite-rich layers (Fig. 4s). Ductile behavior of the coal is also identified by bookshelf sliding with sinistral shear (Fig. 4t) and S–C band structure with dextral shear (Fig. 4u).

Fig. 3. Photographs showing various tectonically deformed coals. (a) Cataclastic coal with claron bands for sample XS7; (b) striated fracture with dull lustre for sample XS7; (c) cataclastic coal with bedding structure for sample XS1; (d) cataclastic coal with angular structure for sample XS9; (e) granulated coal for sample XS5; (f) granulated coal with irregular friction mirror surfaces for sample XS5; (g) lenticular texture with dull lustre of granulated coal for sample XS6; (h) and (i) mylonitic coal with wrinkled texture for samples XS8 and SSP16, respectively.

5.2. Pore size distributions of different TDCs

With pore size larger than 30 nm in cataclastic coals varies from 0.0077 cm³/g to 0.0094 cm³/g, which falls into the range of macropore volume for vitrinite coals in low volatile bituminous rank reported by Gan et al (1972) and Unsworth et al (1989), who adopted macropore size in the range of 30–2960 nm and 30–10,000 nm, respectively. The macro- and mesoporosity in cataclastic coals are low (8.14% to 10.56%), but super-micropore approximately accounts for 85.55% of the total porosity (seen in Table 2), which may be caused by maceral composition.

In comparison to cataclastic coals, the macro- and mesopore volumes of granulated and mylonitic coals sharply increase, and the greatest amount of pore volume in the pore size range of 300–1000 nm (corresponding pressure ranges from 1.2–0.6 MPa) with the highest frequency of pores at diameter about 740 nm is noticed in granulated and mylonitic coals. The results indicate that granulated and mylonitic coals possess six to nine times seepage–pore volume (macro- and mesopore volume) than cataclastic coals, which is consistent with the findings of Li et al. (2003), who applied mercury injection to characterize pore structure of TDCs. However, the super-micropore volume, DR surface area and super-micropore sorption capacity of granulated and mylonitic coals are smaller than those of cataclastic coals, and the adsorption pore volume and surface area of vitrinite-rich coals are higher than those of inertinite-rich coals (Table 2). Moreover, it is found from Fig. 2b that the slopes of volume-pressure curves for granulated and mylonitic coals are highly friable and the HGI is 138 for XS8 and 118 for SSP16, respectively. The SEM observations of these two coals are summarized as follows. Kink band with micro-scale fold occurs in inertinite-rich layers (Fig. 4r) and vitrinite-rich layers (Fig. 4s). Ductile behavior of the coal is also identified by bookshelf sliding with sinistral shear (Fig. 4t) and S–C band structure with dextral shear (Fig. 4u).

Fig. 3. Photographs showing various tectonically deformed coals. (a) Cataclastic coal with claron bands for sample XS7; (b) striated fracture with dull lustre for sample XS7; (c) cataclastic coal with bedding structure for sample XS1; (d) cataclastic coal with angular structure for sample XS9; (e) granulated coal for sample XS5; (f) granulated coal with irregular friction mirror surfaces for sample XS5; (g) lenticular texture with dull lustre of granulated coal for sample XS6; (h) and (i) mylonitic coal with wrinkled texture for samples XS8 and SSP16, respectively.
coals are nearly horizontal when pressure is far beyond 20 MPa (corresponding pore diameter smaller than 60 nm). These characteristics imply that the increasing tectonic deformation mainly leads to the enhancement of seepage–pore volume while have less contribution to the variation in adsorption–pore volume, which was the case in the study of Qu et al. (2010), who found that tectonic deformation mainly reforms the volume of bigger pores (pore size above 100 nm) obtained by MIP method.
According to others’ investigations (Adeloye and Bustin, 2013; Chalmers and Bustin, 2007; Mares et al., 2009), for the iso-rank coals, the maceral type is another important factor affecting pore volume and pore size distribution. Relationship of coal maceral content, macropore volume, mesopore volume, combined trans- and micropore volume and super-micropore volume in the test coal samples was presented in Fig. 6. It is observed that, for the coal samples studied, there is no correlation between the maceral type and macro- and mesopore volume (Fig. 6a and Fig. 6b). However, as shown in Fig. 6c, there is a positive correlation between combined trans- and micropore volume and vitrinite content ($R^2 = 0.975$) and conversely, a negative trend can be observed between combined trans- and micropore volume and inerinite content ($R^2 = 0.975$). The super-micropore volume increases with increasing vitrinite content and decreases with increasing inerinite content except sample XS7 (Fig. 6d). The sample XS7 has the highest super-micropore volume, DR specific surface and the highest adsorbed amount of CO$_2$ which possess a mixture of vitrinite and inerinite (Table 2). The results lie in the findings of Lamberson and Bustin (1993), who suggested that high semifusinite in some coals may create more super-micropore and sorption capacity than vitrinite. These results confirm that the complexity and variability of coal maceral within a narrow range of degree of coalification (1.720% to 1.857%) have predominant effect on the distribution of super- and microporosities (Clarkson and Bustin, 1996). Generally, vitrinite-rich coals have lower macroporosity, higher microporosity and greater sorption capacity than inerinite-rich coals within the iso-rank, which was the case in many studies (Giffin et al., 2013; Prinz et al., 2004).

The variation of pore volume and the relationship between pore volume and maceral composition indicate that tectonic deformation mainly contributes to seepage-porosity and weakens the effect of maceral composition on macro- and mesoporosity. Consequently, poor correlation occurs between maceral composition and macroporosity and mesoporosity. Similar results also can be found in the work of Li et al. (2003). However, maceral type mainly determines the distribution of adsorption-pores, while tectonic deformation has no or very little effect on the variation of adsorption-pores among the coals studied here. The increment of macro- and mesoporosity in granulated and mylonitic coals may be attributed to the presence of microfracture (aperture lower than 10 μm) and inter-granular pores formed during deformation process. The coal particles form tiny clasts under intensive shear deformation as depicted in granulated and mylonitic coals in Section 5.1. A dense cluster of microfracture and inter-granular pores with pore size mainly less than 5 μm form between these clasts (Fig. 4k, l, p, v, w), which corresponds to the increasing range of seepage-pores. Zhang (2001) also found that the size of inter-granular pore varies from 0.5 μm to 5 μm falling into the range of meso- and macropore sizes based on the observation of SEM, which confirmed our results. Therefore, the developed microfracture and inter-granular pores are responsible for the increase of seepage-pore volume under increasing tectonic deformation.

### 5.3. Multifractal analysis of Hg PSDs of different coals

The double log plots of partition functions, $\chi(q, c)$, versus box size, $c$, estimated from Eq. (4) are built for box size range $c \in [L 	imes 2^{-k}, 1]$ ($0 \leq k \leq 6$) and moment range $q \in [-10, 10]$. Fig. 7 presented two select-ed plots of log $\chi(q, c)$-log$c$ for Hg PSDS with the best (XS7) and the worst (XS6) linear correlation. If $\chi(q, c)$ versus $c$ obeys a power law scaling or a linear relationship can be observed between log $\chi(q, c)$ and log$c$, the distribution of pore sizes in porous media is considered a multifractal distribution (Muller, 1996). All coal samples show a good linear relationship between log $\chi(q, c)$ and log$c$ with coefficients of determination, $R^2$, larger than 0.96 for all values of $q$ (Fig. 7). This illustrates that the PSDs of coals have multifractal characteristics. Thus, it is necessary to explore the inner variations in PSDs of different coal samples.

Generalized dimensions, $D_q$, calculated from Eqs. (5) and (6) for different coals were listed in Table 4 and the $D_q$ spectra were presented in Fig. 8. From Fig. 4, it is found that all the spectra follow a monotone decreasing function of $q$ with a sigma-shaped curve. Moreover, from Table 4, the three dimensions for all samples follow the order as: $D_0 > D_1 > D_2$. The shape of the $D_q$ spectra and the variation of $D_q$ values reveal that the PSDs of coal samples have properties close to multifractal self-similarity measures, which is in good agreement with the observation from $\chi(q, c)$ presented above.

The shape and parameters of $D_q$ spectra including information dimension $D_0$, the Hurst exponent $H$, the width $D_{-10}-D_{10}$ of $D_q$ spectrum, the width of right side $D_{-10}-D_{0}$ of $D_q$ spectrum and the width of left side $D_{10}-D_{0}$ of $D_q$ spectrum listed in Table 4 give information about the inner variability in the distribution of pore sizes and also depict the heterogeneity in size-dependent distribution of porosity (Paz Ferriero and Vidal Vázquez, 2010). The wider the $D_q$ spectrum, the higher is the complexity or heterogeneity in PSDs (Vidal Vázquez et al., 2008).

The narrowest $D_q$ spectrum or the smallest value of $D_{-10}-D_{10}$, indicative of the lowest degree of heterogeneity in the distribution of porosity along the pore size intervals, can be observed in sample XS7 (Table 4). The widest distributions of $D_q$ spectrum is found in sample XS6 (Table 4), indicating the highest degree of multifractality or inhomogeneity in PSDS. However, the $D_{-10}-D_{10}$ values do not always grow with increasing tectonic deformation. For example, the value of $D_{-10}-D_{10}$ of sample XS9 is almost close to XS5 but larger than sample XS8 (Table 4). This may be caused by the joint action of factors that affect the pore volume and size-distribution of pores. The variation of PSDS may be related to coal rank, maceral type, moisture, carbon and ash content and tectonic deformation (Clarkson and Bustin, 1996; Giffin et al., 2013; Hou et al., 2012; Li et al., 2003). Since the variation of width $D_{-10}-D_{10}$ of $D_q$ spectrum reflects the heterogeneity in the distribution of porosity over the whole pore size range, it may be the comprehensive result of these factors and cannot completely reflect the main distinction in PSDs for coals with different structure.

However, significant differences exist in the shape variation of $D_q$ versus $q$ for cataclastic coals, the $D_q$ spectra show a quasi-linear variation for $q > 0$, whereas a sigma-shaped curve for $q < 0$ even if it is not very obvious in sample XS7 (Fig. 8). On the contrary, the variations of $D_q$ with respect to $q$ exhibit a reverse trend in granulated and mylonitic coals. The $D_q$ spectra of samples XS5, XS6, XS8 and SSP16 show a rather sigma-shaped curve for $q = 0$, whereas a quasi-linear variation $D_q$ versus $q$ for $q < 0$ is observed. These variations of $D_q$ with respect to $q$ also can be confirmed by the comparison of widths of left side $D_{-10}-D_{0}$ and right side $D_{0}-D_{10}$ of $D_q$ spectra (Table 4). From Table 4, in granulated and mylonitic coals, the $D_{0}-D_{10}$ values are larger than the values of $D_{-10}-D_{0}$ and the difference is higher than 0.1, whereas negative trend can be observed in cataclastic coals.

The variation of the right part $D_{0}-D_{10}$ for $q > 0$ and left part $D_{-10}-D_{0}$ for $q < 0$ of $D_q$ spectra corresponds to dominance of large and small concentration of porosity, respectively (Caniego et al., 2003; Paz Ferriero and Vidal Vázquez, 2010). Consequently, the change of multifractal parameters for $q > 0$ may be due to various distributions of pore size larger than 100 nm (seepage-pores) and parameters for $q < 0$ can be assigned to pore size smaller than 100 nm (adsorption-pores) as presented in Fig. 5. Quasi-linear shape stands for a homogeneous distribution of pore sizes, whereas sigma-shaped curve stands for a heterogeneous distribution of pore sizes (Caniego et al., 2003). Hence, cataclastic coals show a homogeneous structure in the inner distribution of seepage-pores but a heterogeneous structure in the inner distribution of adsorption-pores, while the opposite trend is noticed in granulated and mylonitic coals.

Additionally, granulated and mylonitic coals have more concave up shape for $q > 0$ and greater widths of right side $D_{0}-D_{10}$ than cataclastic coals, which indicate that tectonic deformation increases the variability and heterogeneity in the inner distribution of seepage-pores. However, it is not a common phenomenon in the widths of left side $D_{-10}-D_{0}$ of $D_q$ spectra ($q < 0$), as the values of $D_{-10}-D_{0}$ in some cataclastic coals are
wider than those in granulated and mylonitic coals. For example, the sample XS9 characterized by the highest vitrinite content has the greatest \(D_{-10} - D_{0}\) value, while the smallest \(D_{-10} - D_{0}\) value occurs in sample SSP16 represented by the greatest inertinite content. This phenomenon indicates that the right branch of \(D_q\) spectra mainly reflects the influence of tectonic deformation on the PSDs of coals and maceral type exerts important effect on the variation of left side \(D_{-10} - D_{0}\) of \(D_q\) spectra. The above analysis is compatible with the results of PSDs in Section 5.2 that tectonic deformation and maceral composition exert primary control on the distribution of seepage-pores and adsorption-pores, respectively.

The capacity dimension, \(D_0\), represents the scaling of non-empty boxes containing some porosity under successive finer partitions, which is independent of the probability of the porosity in each box (Caniego et al., 2003; Muller and McCauley, 1992). From Table 5, the values of \(D_0\) are always 1.000 for all samples, which corresponds to the Euclidean dimension of one for one-dimensional distribution. This is probably due to the fact that, when all the boxes have some concentration of porosity, the partition function \(\chi_{\epsilon}(0, \epsilon)\) in Eq. (4) equals the total number of boxes \(N(\epsilon)\) covering the pore size length, so \(D_0\) shown in Eq. (5) can be expressed as \(D_0 = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log \epsilon}\), which scales with Euclidean exponent one. This result is consistent with the conclusion of other scholars (Paz Ferreiro and Vidal Vázquez, 2010; San José Martínez et al., 2010; Vidal Vázquez et al., 2008).

The information dimension, \(D_1\), provides information about the concentration degree of distribution of porosity along pore size intervals (Vidal Vázquez et al., 2008). The maximum value of \(D_1\) is \(D_0\) which occurred in monofractal PSDs (Paz Ferreiro and Vidal Vázquez, 2010). The values of \(D_1\) are closer to \(D_0\), the porosity is more evenly distributed across the range of pore sizes. And conversely the smaller the values of \(D_1\), the more clustered the pores, the higher the porosity is concentrated in a small domain of the study scale and the higher degree of heterogeneity occurs in inner PSDs (Caniego et al., 2003). Table 5 showed that the values of \(D_1\) for cataclastic coals vary from 0.966 to 0.974, rather close to \(D_0\), an indicator of a rather homogeneous distribution of
porosity over the range of pore sizes. In contrast, the lower $D_1$ values (0.860 to 0.896) of granulated and mylonitic coals suggest increased clustering of PSDs, so that most of the porosity concentrates in a narrow range of equivalent diameter sizes and higher degree of unevenness of inner PSDs occurs. These results are compatible with a relatively regular multi-model PSDs of cataclastic coals in Fig. 5, which contrast with narrow and high fluctuant PSDs of granulated and mylonitic coals in Fig. 5. It is observed that the $D_1$ increases as the porosity in pore size above 100 nm increases due to increasing tectonic deformation. This implies that the increment in seepage-porosity, mainly in 1.0 above 100 nm increases due to increasing tectonic deformation. This multi-model PSDs of cataclastic coals in Fig. 5, which contrast with inner PSDs occurs. These results are compatible with a relatively regular clustering of PSDs, so that most of the porosity concentrates in a narrow distributions of seepage-pores of different coals.

The Hurst exponent, $H$, indicates the autocorrelation of distribution of porosity over the set of pore sizes related to long-range dependencies (San José Martínez et al., 2010). The value of $H$ closer to 1 means that the stronger autocorrelation does exist in size-dependent distribution of porosity (San José Martínez et al., 2010). The mean values of $H$ for cataclastic, granulated and mylonitic coal samples are 0.971, 0.873 and 0.893, respectively (Table 5). All the values of $H$ for different coal samples are close to 1, indicating positive autocorrelation in the variation of porosity among different pore size intervals. Although all the values of $H$ are close to 1, differences are still found in the values of $H$ in different coals. The mean $H$ values of cataclastic coals are significantly larger than those of granulated and mylonitic coals, indicative of a higher autocorrelation in size-dependent distribution of porosity of cataclastic coals. This phenomenon also shows that the lower pore connectivity along the interval of pore sizes occurs with increasing tectonic deformation. Thus, $H$ can be considered as an important parameter for estimating the evolution of pore connectivity across the range of pore sizes after increased tectonic deformation.

Previous studies show that the conventional fractal dimension (CFD) is an effective parameter to describe the geometrical complexity of PSDs of coals (Mahamud et al., 2003; Yao et al., 2009a; Zou et al., 2013). Since both CFD and multifractal parameters can be used to characterize the heterogeneity of PSDs, a correlation may exist between them. Here we investigated the relationship between these parameters. Three main mathematical models for CFD related to Hg PSDs were proposed by Friesen and Mikula (1987), Neimark (1990) and Zhang et al. (2006). In this study, the Zhang’s model was chosen to determine the fractal dimension $D_2$ of PSDs due to its lower deviations and higher correlation coefficients (Zhang and Li, 1995). A detailed description of the procedure of fractal theories developed by Zhang and coworkers (2006) can be found in our previous research (Zheng et al., 2015) where the method was already employed.

Fig. 9 showed the plots of $\ln(W_{z,\epsilon}/n_r)$ versus $\ln(V_{n}^{1/3}/n_r)$ for the test coals. Each graph includes the average fractal dimension $D_2$ for the full range of pores. As presented in Fig. 9, for the coal samples studied, no obvious difference exists in $D_2$ of different TDCs with different maceral
compositions. That is, the average fractal dimension $D_q$ cannot discriminate between PSDs of TDCs. Under this situation, we calculated CFD $D_{ap}$ and $D_{sp}$ corresponding to seepage-pore and adsorption-pore respectively as suggested by the variations of $D_0 - D_{10}$ for $q > 0$ and $D_{-10} - D_0$ for $q < 0$ of $D_q$ spectra. The results were shown in Table 4. It is observed that increasing tectonic deformation shows a trend to increase $D_{sp}$, while increasing vitrinite causes increasing $D_{ap}$. This is in line with the observation from $D_0 - D_{10}$ and $D_{-10} - D_0$ presented above. The results suggest that different pore size intervals show different self-similarities (multifractal scaling), so the average fractal dimension $D_q$ over the whole pore size range is failed to characterize the variability of PSDs of coal samples with different deformation structures and petrographic compositions.

Linear regression analysis between CFD ($D_{sp}$, $D_{ap}$) and multifractal parameters was presented in Table 5. As shown in Table 5, $D_{ap}$ is positively correlated with $D_{sp}$ and $D_0$, and negatively correlated with $D_0$ and $H$. $D_{ap}$ exhibits a positive trend with $D_{-10}$-$D_0$. The positive or negative correlation between $D_{sp}$ or $D_{ap}$ and multifractal parameters can be explained by the reason that these parameters reflect similar pore scaling properties dominating at different pore size intervals. However, multifractal analysis provides more parameters to quantitatively express the inner scaling behavior of PSDs of coal compared with conventional single fractal analysis. Since $D_{sp}$ and the generalized dimension $D_{-10}$-$D_0$ characterize different scaling behavior of PSDs, the correlation between them is not analyzed, as well as $D_{sp}$ and multifractal parameters for $q > 0$.

The above analysis suggests that scaling property or multifractality in PSDs of coals may depend on coal rank, maceral type, carbon content, ash content and tectonic deformation. The variation of multifractal scaling in Hg PSDs of coals with the same rank in our study may greatly lie on tectonic deformation and maceral type. Therefore, the various factors, dominating at different pore size intervals, show specific influence on multifractal scaling in the distribution of pore sizes measured by Hg injection.

5.4. Relationship of porosity, maceral composition and multifractal parameters from Hg PSDs

Relationship of multifractal parameters, porosity (macro-, meso-, seepage- and combined trans- and microporosity) and maceral composition was performed through a linear regression analysis (Table 5). The results show that macro-porosity and seepage-porosity are significantly and positively correlated to $D_0 - D_{10}$, but negatively correlated to $D_0$ and $H$ at the $p = 0.01$ level ($R > 0.85$). Mesoporosity shows similar correlation to above multifractal parameters for moment $q > 0$ at the $p = 0.01$ level with the exception of $D_0 - D_{10}$ (significant at the $p = 0.05$ level and $R = 0.814$). The macro-porosity also positively correlates to $D_{-10} -$ $D_0$ at the $p = 0.1$ level, but the correlation coefficient is somewhat lower ($R = 0.666$). In contrast, combined trans- and microporosity show a positive correlation to multifractal parameter $D_{-10} - D_0$ for moment $q < 0$ at the $p = 0.05$ level, but exhibit lower correlation to multifractal parameters for $q > 0$ ($R$ ranging from $-0.389$ to $0.436$). The correlation between coal composition and multifractal parameters follows the similar tendency as combined trans- and microporosity. Vitrinite and inertinite shows a positively and negatively weak trend with $D_{-10} - D_0$ (significant at the $p = 0.1$ level) respectively but show no or little correlation to multifractal parameters for moment $q < 0$. It is worth noting that no clear correlation is found between moisture content, ash content, carbon content and multifractal parameters (data not shown). Thus, the effect of the aforementioned factors on multifractal scaling in Hg PSDs can be neglected.

The strong relationship between macro-, meso-, and seepage-porosity and multifractal parameters (e.g., $D_0$, $H$) for $q > 0$ can be interpreted as the fact that these parameters depict the heterogeneity and distribution of high concentrations of porosity (seepage-pore in our study) in Hg PSDs that are magnified by positive values of $q$. In contrast, the high correlation between trans-microporosity, maceral composition and multifractal parameter $(D_{-10} - D_0)$ for $q < 0$ are

![Image of multifractal parameters from the generalized dimension spectrum and conventional fractal dimensions for the test coal samples.](image)

**Table 4**
Selected multifractal parameters from the generalized dimension spectrum and conventional fractal dimensions for the test coal samples.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$H$</th>
<th>$D_{10}$</th>
<th>$D_{-10}$</th>
<th>$D_{sp}$</th>
<th>$D_{ap}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>XS7</td>
<td>1.00</td>
<td>0.974</td>
<td>0.977</td>
<td>0.899</td>
<td>1.202</td>
<td>0.303</td>
<td>0.202</td>
</tr>
<tr>
<td>XS1</td>
<td>1.00</td>
<td>0.964</td>
<td>0.968</td>
<td>0.837</td>
<td>1.320</td>
<td>0.483</td>
<td>0.163</td>
</tr>
<tr>
<td>XS9</td>
<td>1.00</td>
<td>0.966</td>
<td>0.967</td>
<td>0.793</td>
<td>1.393</td>
<td>0.600</td>
<td>0.207</td>
</tr>
<tr>
<td>XS5</td>
<td>1.00</td>
<td>0.885</td>
<td>0.891</td>
<td>0.625</td>
<td>1.230</td>
<td>0.604</td>
<td>0.375</td>
</tr>
<tr>
<td>XS6</td>
<td>1.00</td>
<td>0.860</td>
<td>0.871</td>
<td>0.566</td>
<td>1.300</td>
<td>0.734</td>
<td>0.434</td>
</tr>
<tr>
<td>XS8</td>
<td>1.00</td>
<td>0.890</td>
<td>0.901</td>
<td>0.652</td>
<td>1.216</td>
<td>0.564</td>
<td>0.348</td>
</tr>
<tr>
<td>SSP16</td>
<td>1.00</td>
<td>0.896</td>
<td>0.886</td>
<td>0.547</td>
<td>1.164</td>
<td>0.618</td>
<td>0.453</td>
</tr>
</tbody>
</table>

$D_0$ and $D_1$ are the capacity and information dimensions, respectively; $H$, the Hurst exponent; $D_{10}$ and $D_{-10}$ are the generalized dimensions for moment $q = 10$ and $q = -10$, respectively; $D_{sp}$-$D_{10}$ is the width of $D_{sp}$ spectrum; $D_{ap}$-$D_{10}$ and $D_{sp}$-$D_{ap}$ are the width of right side and left side of $D_{sp}$ spectrum, respectively; $D_{ap}$ and $D_{sp}$ are the conventional fractal dimensions corresponding to seepage-pore and adsorption-pore, respectively.

**Table 5**
Relationship of conventional fractal dimension $D_{sp}$ and $D_{ap}$, porosity, maceral composition, and multifractal parameters from Hg PSDs.

<table>
<thead>
<tr>
<th>$D_0$</th>
<th>$H$</th>
<th>$D_{-10} - D_{10}$</th>
<th>$D_{ap}$</th>
<th>$D_{sp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{ap}$</td>
<td>$-0.683$</td>
<td>$-0.781$</td>
<td>$0.307$</td>
<td>$0.833$</td>
</tr>
<tr>
<td>$D_{ap}$</td>
<td>$n$</td>
<td>$n$</td>
<td>$nc$</td>
<td>$n$</td>
</tr>
<tr>
<td>Macroporosity</td>
<td>$0.972$</td>
<td>$0.955$</td>
<td>$0.666$</td>
<td>$0.877$</td>
</tr>
<tr>
<td>Mesoporosity</td>
<td>$0.901$</td>
<td>$0.890$</td>
<td>$0.461$</td>
<td>$0.814$</td>
</tr>
<tr>
<td>Seepage-porosity</td>
<td>$0.940$</td>
<td>$0.937$</td>
<td>$0.557$</td>
<td>$0.858$</td>
</tr>
<tr>
<td>Trans + microporosity</td>
<td>$0.436$</td>
<td>$0.398$</td>
<td>$0.305$</td>
<td>$0.389$</td>
</tr>
<tr>
<td>Vitrinite</td>
<td>$0.314$</td>
<td>$0.111$</td>
<td>$nc$</td>
<td>$-0.203$</td>
</tr>
<tr>
<td>Inertinite</td>
<td>$-0.312$</td>
<td>$-0.094$</td>
<td>$nc$</td>
<td>$0.221$</td>
</tr>
</tbody>
</table>

*, significant at the 0.05 probability ($p$) level; †, significant at the $p = 0.01$ level; ††, significant at the $p = 0.1$ level; nc, means no correlation is observed; n means not analyzed.
Fig. 9. Plots of $\ln(W_n/r_n^2)$ versus $\ln(V_n^{1/3}/r_n)$ for the test coal samples. The value of $D_z$ is the average fractal dimension.
attributed to the parameter describing the variability and distribution of small concentrations of porosity (adsorption-pore in our study) in Hg PSDs that are magnified by negative values of $q$. However, there is little multifractal parameter to characterize the multifractal properties of adsorption-pores. Hence, it is necessary to fully characterize the size-dependent distribution of adsorption-porosity and the work focused on the multifractal analysis of PSDs based on nitrogen isotherms is currently being done to quantitatively explore how maceral type and other factors (e.g., ash content) affects the multifractality and important physical properties of inner size-distributions of adsorption-pores.

The above regression results suggest that, for the coals studied, the increasing macro- and mesoporosity or seepage-porosity caused by tectonic deformation induces higher clustering, lower autocorrelation, higher heterogeneity as estimated by generalized dimensions for $q > 0$. Therefore, the narrower distribution with higher fluctuation, lower pore connectivity and greater complexity in the inner distribution of seepage-pores occur in granulated and mylonitic coals when compared with cataclastic coals. The increasing seepage-porosity may relate to the presence of microfracture and inter-granular pores formed through deformation process as depicted in Section 5.2. Consequently, the uneven distribution of microfracture and inter-granular pores caused by tectonic deformation may be the main factors that affect the variation of generalized dimensions for $q < 0$ and induce a more heterogeneous distribution of seepage-porosity along the interval of pore sizes. Moreover, the strong correlation means that $D_1$, $H$, and $D_{10} - D_{10}$ can be treated as the predictive parameters that well discriminate between Hg PSDs of TDCs. However, the increment in combined trans- and microporosity due to increasing vitrinite leads to high heterogeneous distributions of adsorption-pores, which can be reflected by the positive relationship of trans-microporosity, maceral composition and multifractal parameter $D_{10} - D_{10}$ for $q > 0$.

In summary, in comparison to single pore volume, this investigation shows that the multifractal parameters, such as $D_1$, $H$ and $D_{10} - D_{10}$, can well describe the inner distribution of porosity along pore size intervals and distinguish the internal differences in Hg PSDs hidden in the pore volume–size data series for different coals in more detail. The multifractal analysis generates more useful parameters which can well elucidate how tectonic deformation affects the evolution and heterogeneity of Hg PSDs of coals.

6. Conclusions

In this work, in combination with mercury intrusion data and gas ($N_2$ and $CO_2$) adsorption data, the Hg pore volume under pressure > 10 MPa was corrected, and then the PSDs related to coal deformation structures after Hg data correction were investigated. The results show that, under the iso-rank conditions, tectonic deformation has a dramatic effect on seepage-porosity but has less contribution to the variation in adsorption-porosity for the coals studied. In contrast, the maceral type is the main control upon the distribution of adsorption-porosity especially for super-microporosity but has little contribution to the distribution of seepage-porosity due to the effect of tectonic deformation. Generally, seepage-porosity (macro- and mesoporosity) increases with increasing tectonic deformation, which may be attributed to the presence of microfractures and inter-granular pores formed during deformation process. Vitrinite-rich coals have greater amount of adsorption-porosity than inertinite-rich coals.

The shape of generalized spectra and the variation of generalized dimensions $D_q$ with respect to $q$ reveal that the PSDs of coal samples show multifractal behavior. Nevertheless, the extracted generalized dimensions from PSDs including information dimension $D_q$, the Hurst exponent $H$, the width of right side $D_{10} - D_{10}$ of $D_q$ spectra for moment $q > 0$ and the width of left side $D_{10} - D_{10}$ of $D_q$ spectra for moment $q < 0$ exist significantly difference in the coals studied. The variation of $D_1$, $H$, and $D_{10} - D_{10}$ for $q > 0$ and a strong correlation between these parameters and seepage-porosity illustrate that tectonic deformation leads to narrower distribution with higher fluctuation, lower pore connectivity and greater complexity in the inner distribution of seepage-pores. On the contrary, the $D_{10} - D_{10}$ values for $q < 0$ and high correlation with adsorption-porosity suggest that increasing vitrinite content induces a heterogeneous structure in the inner distribution of adsorption-pores. Moreover, $D_1$, $H$, and $D_{10} - D_{10}$ can well discriminate between PSDs of TDCs. Hence, multifractal approach is useful to characterize the internal heterogeneity and amplify the differences in PSDs of different coals.

A further step is needed to correlate such generalized dimensions to physical properties of TDCs, such as gas properties or permeability and mechanical properties or friability. Multifractal analysis may be a useful way to quantify these behaviors. Moreover, it is still need to further investigate how other factors, including coal rank and maceral composition, affect multifractal properties of distribution of adsorption-pores for coals.

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